11.4.1 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 11 materials.

In Exercises 1 - 16, plot the point given in polar coordinates and then give three different expressions for the point such that (a) $r < 0$ and $0 \leq \theta \leq 2\pi$, (b) $r > 0$ and $\theta \leq 0$ (c) $r > 0$ and $\theta \geq 2\pi$

For help with these exercises, click the resource below:

- Introduction to polar coordinates

1. $\left(2, \frac{\pi}{3}\right)$
2. $\left(5, \frac{7\pi}{4}\right)$
3. $\left(\frac{1}{3}, \frac{3\pi}{2}\right)$
4. $\left(\frac{5}{2}, \frac{5\pi}{6}\right)$
5. $\left(12, -\frac{7\pi}{6}\right)$
6. $\left(3, -\frac{5\pi}{4}\right)$
7. $(2\sqrt{2}, -\pi)$
8. $\left(\frac{7}{2}, -\frac{13\pi}{6}\right)$
9. $(-20, 3\pi)$
10. $\left(-4, \frac{5\pi}{4}\right)$
11. $\left(-1, \frac{2\pi}{3}\right)$
12. $\left(-3, \frac{\pi}{2}\right)$
13. $\left(-3, -\frac{11\pi}{6}\right)$
14. $\left(-2.5, -\frac{\pi}{4}\right)$
15. $\left(-\sqrt{5}, -\frac{4\pi}{3}\right)$
16. $(-\pi, -\pi)$

In Exercises 17 - 36, convert the point from polar coordinates into rectangular coordinates.

For help with these exercises, click the resource below:

- Converting between rectangular and polar coordinates

17. $\left(5, \frac{7\pi}{4}\right)$
18. $\left(2, \frac{\pi}{3}\right)$
19. $\left(11, -\frac{7\pi}{6}\right)$
20. $(-20, 3\pi)$
21. $\left(\frac{3}{5}, \frac{\pi}{2}\right)$
22. $\left(-4, \frac{5\pi}{6}\right)$
23. $\left(9, \frac{7\pi}{2}\right)$
24. $\left(-5, -\frac{9\pi}{4}\right)$
25. $\left(42, \frac{13\pi}{6}\right)$
26. $(-117, 117\pi)$
27. $(6, \arctan(2))$
28. $(10, \arctan(3))$
29. $\left(-3, \arctan\left(\frac{4}{3}\right)\right)$
30. $\left(5, \arctan\left(-\frac{4}{3}\right)\right)$
31. $\left(2, \pi - \arctan\left(\frac{1}{2}\right)\right)$
32. $\left(-\frac{1}{2}, \pi - \arctan(5)\right)$
33. $\left(-1, \pi + \arctan\left(\frac{3}{4}\right)\right)$
34. $\left(\frac{2}{3}, \pi + \arctan\left(2\sqrt{2}\right)\right)$
35. $(\pi, \arctan(\pi))$
36. $\left(13, \arctan\left(\frac{12}{5}\right)\right)$

In Exercises 37 - 56, convert the point from rectangular coordinates into polar coordinates with $r \geq 0$ and $0 \leq \theta < 2\pi$.

For help with these exercises, click the resource below:

- Converting between rectangular and polar coordinates

37. $(0, 5)$
38. $(3, \sqrt{3})$
39. $(7, -7)$
40. $(-3, -\sqrt{3})$
41. $(-3, 0)$
42. $\left(-\sqrt{2}, \sqrt{2}\right)$
43. $\left(-4, -4\sqrt{3}\right)$
44. $\left(\frac{\sqrt{3}}{4}, -\frac{1}{4}\right)$
45. $\left(-\frac{3}{10}, -\frac{3\sqrt{3}}{10}\right)$
46. $\left(-\sqrt{5}, -\sqrt{5}\right)$
47. $(6, 8)$
48. $(\sqrt{5}, 2\sqrt{5})$
49. $(-8, 1)$
50. $(-2\sqrt{10}, 6\sqrt{10})$
51. $(-5, -12)$
52. $\left(-\frac{\sqrt{5}}{15}, -\frac{2\sqrt{5}}{15}\right)$
53. $(24, -7)$
54. $(12, -9)$
55. $\left(\frac{\sqrt{2}}{4}, \frac{\sqrt{6}}{4}\right)$
56. $\left(-\frac{\sqrt{65}}{5}, \frac{2\sqrt{65}}{5}\right)$

In Exercises 57 - 76, convert the equation from rectangular coordinates into polar coordinates. Solve for r in all but #60 through #63. In Exercises 60 - 63, you need to solve for θ

For help with these exercises, click the resource below:

- Converting rectangular equations to polar equations

57. $x = 6$
58. $x = -3$
59. $y = 7$
60. $y = 0$
61. $y = -x$
62. $y = x\sqrt{3}$
63. $y = 2x$
64. $x^2 + y^2 = 25$
65. $x^2 + y^2 = 117$
66. $y = 4x - 19$
67. $x = 3y + 1$
68. $y = -3x^2$
69. $4x = y^2$
70. $x^2 + y^2 - 2y = 0$
71. $x^2 - 4x + y^2 = 0$
72. $x^2 + y^2 = x$
73. $y^2 = 7y - x^2$
74. $(x + 2)^2 + y^2 = 4$
75. $x^2 + (y - 3)^2 = 9$
76. $4x^2 + 4\left(y - \frac{1}{2}\right)^2 = 1$

In Exercises 77 - 96, convert the equation from polar coordinates into rectangular coordinates. For help with these exercises, click the resource below:

- Converting polar equations to rectangular equations

77. $r = 7$
78. $r = -3$
79. $r = \sqrt{2}$
80. $\theta = \dfrac{\pi}{4}$
81. $\theta = \dfrac{2\pi}{3}$
82. $\theta = \pi$
83. $\theta = \dfrac{3\pi}{2}$
84. $r = 4\cos(\theta)$
85. $5r = \cos(\theta)$
86. $r = 3\sin(\theta)$
87. $r = -2\sin(\theta)$
88. $r = 7\sec(\theta)$
89. $12r = \csc(\theta)$
90. $r = -2\sec(\theta)$
91. $r = -\sqrt{5}\csc(\theta)$
92. $r = 2\sec(\theta)\tan(\theta)$
93. $r = -\csc(\theta)\cot(\theta)$
94. $r^2 = \sin(2\theta)$
95. $r = 1 - 2\cos(\theta)$
96. $r = 1 + \sin(\theta)$

97. Convert the origin $(0, 0)$ into polar coordinates in four different ways.
98. With the help of your classmates, use the Law of Cosines to develop a formula for the distance between two points in polar coordinates.

Checkpoint Quiz 11.4

1. Convert the following from rectangular coordinates to polar coordinates. Check your answers.

   (a) $P(2\sqrt{3}, -2)$ (b) $P(1, 3)$

2. Convert the equation $r = \dfrac{1}{2 + \cos(\theta)}$ in polar coordinates into an in equation rectangular coordinates.

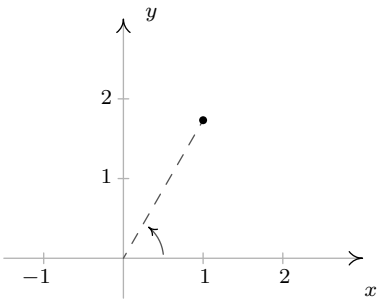
3. Convert the equation $x^2 + y^2 - 6y = 0$ in rectangular coordinates into an in equation polar coordinates.

For worked out solutions to this quiz, click the links below:

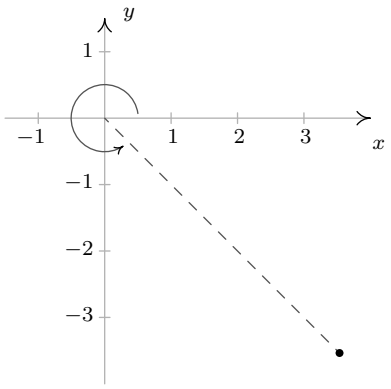
- Solution Part 1
- Solution Part 2

11.4.2 ANSWERS

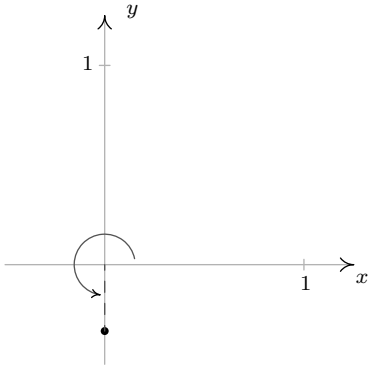
1. $\left(2, \frac{\pi}{3}\right), \left(-2, \frac{4\pi}{3}\right)$
   $\left(2, -\frac{5\pi}{3}\right), \left(2, \frac{7\pi}{3}\right)$



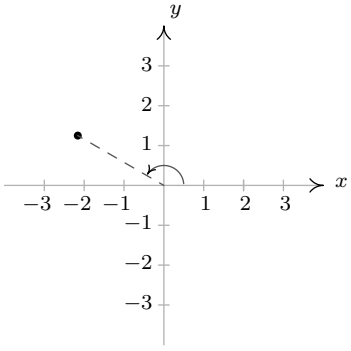
2. $\left(5, \frac{7\pi}{4}\right), \left(-5, \frac{3\pi}{4}\right)$
   $\left(5, -\frac{\pi}{4}\right), \left(5, \frac{15\pi}{4}\right)$



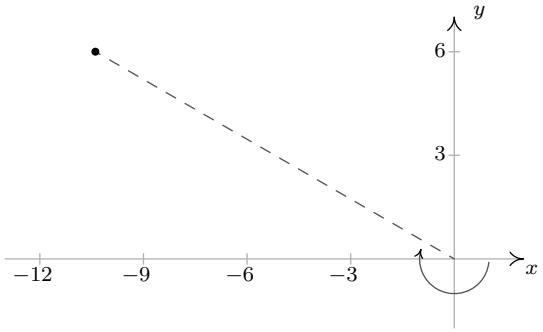
3. $\left(\frac{1}{3}, \frac{3\pi}{2}\right), \left(-\frac{1}{3}, \frac{\pi}{2}\right)$
   $\left(\frac{1}{3}, -\frac{\pi}{2}\right), \left(\frac{1}{3}, \frac{7\pi}{2}\right)$



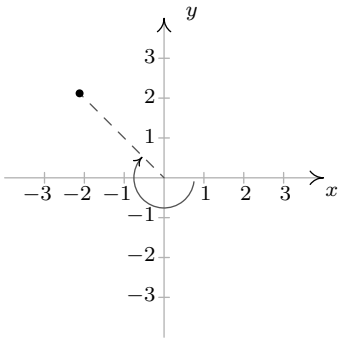
4. $\left(\frac{5}{2}, \frac{5\pi}{6}\right), \left(-\frac{5}{2}, \frac{11\pi}{6}\right)$
   $\left(\frac{5}{2}, -\frac{7\pi}{6}\right), \left(\frac{5}{2}, \frac{17\pi}{6}\right)$



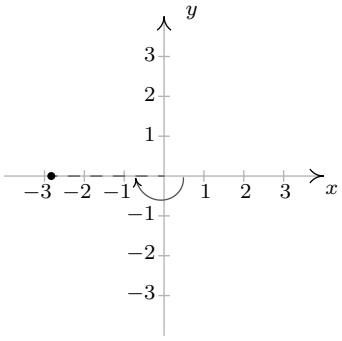
5. $\left(12, -\frac{7\pi}{6}\right), \left(-12, \frac{11\pi}{6}\right)$
$\left(12, -\frac{19\pi}{6}\right), \left(12, \frac{17\pi}{6}\right)$



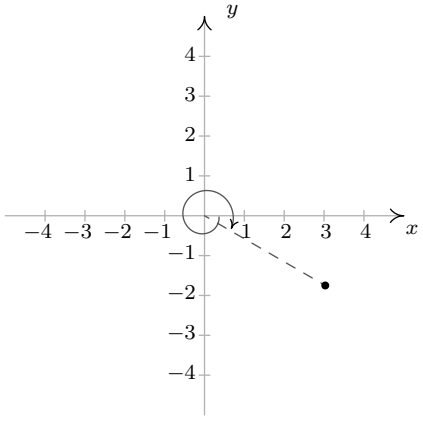
6. $\left(3, -\frac{5\pi}{4}\right), \left(-3, \frac{7\pi}{4}\right)$
$\left(3, -\frac{13\pi}{4}\right), \left(3, \frac{11\pi}{4}\right)$



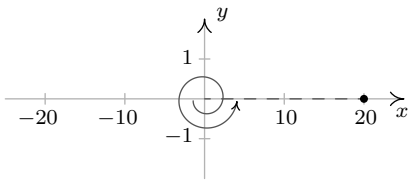
7. $\left(2\sqrt{2}, -\pi\right), \left(-2\sqrt{2}, 0\right)$
$\left(2\sqrt{2}, -3\pi\right), \left(2\sqrt{2}, 3\pi\right)$



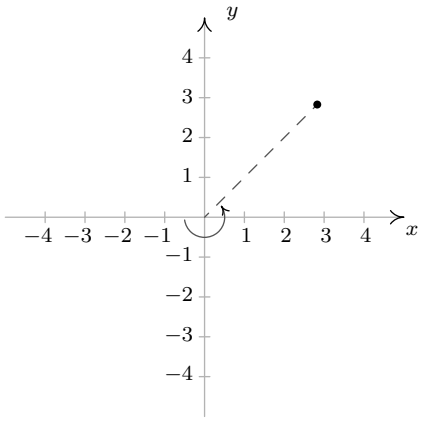
8. $\left(\frac{7}{2}, -\frac{13\pi}{6}\right), \left(-\frac{7}{2}, \frac{5\pi}{6}\right)$
$\left(\frac{7}{2}, -\frac{\pi}{6}\right), \left(\frac{7}{2}, \frac{23\pi}{6}\right)$



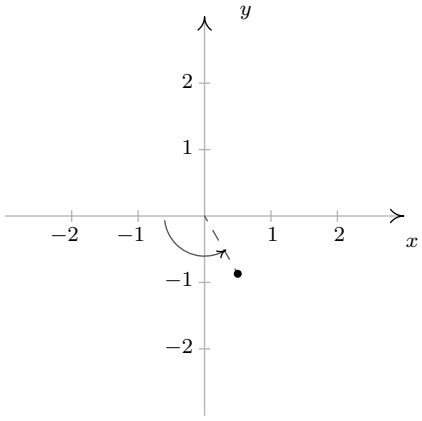
9. $(-20, 3\pi)$, $(-20, \pi)$
$(20, -2\pi)$, $(20, 4\pi)$



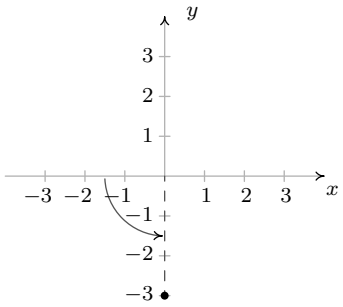
10. $\left(-4, \frac{5\pi}{4}\right)$, $\left(-4, \frac{13\pi}{4}\right)$
$\left(4, -\frac{7\pi}{4}\right)$, $\left(4, \frac{9\pi}{4}\right)$



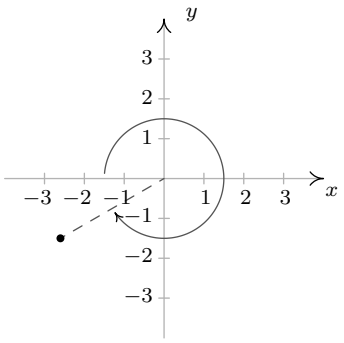
11. $\left(-1, \frac{2\pi}{3}\right)$, $\left(-1, \frac{8\pi}{3}\right)$
$\left(1, -\frac{\pi}{3}\right)$, $\left(1, \frac{11\pi}{3}\right)$



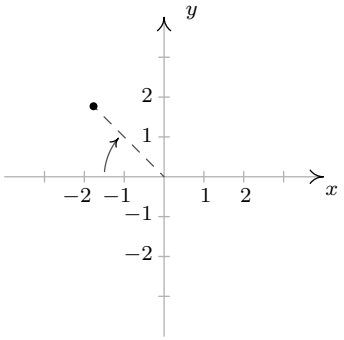
12. $\left(-3, \frac{\pi}{2}\right)$, $\left(-3, \frac{5\pi}{2}\right)$
$\left(3, -\frac{\pi}{2}\right)$, $\left(3, \frac{7\pi}{2}\right)$



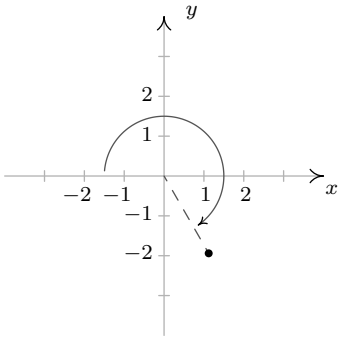
13. $\left(-3, -\frac{11\pi}{6}\right)$, $\left(-3, \frac{\pi}{6}\right)$
$\left(3, -\frac{5\pi}{6}\right)$, $\left(3, \frac{19\pi}{6}\right)$



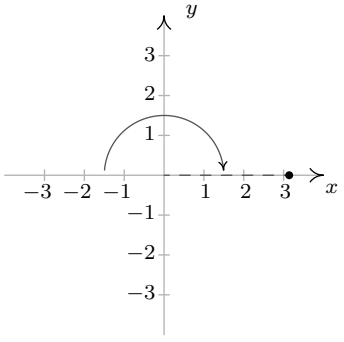
14. $\left(-2.5, -\frac{\pi}{4}\right)$, $\left(-2.5, \frac{7\pi}{4}\right)$
$\left(2.5, -\frac{5\pi}{4}\right)$, $\left(2.5, \frac{11\pi}{4}\right)$



15. $\left(-\sqrt{5}, -\frac{4\pi}{3}\right)$, $\left(-\sqrt{5}, \frac{2\pi}{3}\right)$
$\left(\sqrt{5}, -\frac{\pi}{3}\right)$, $\left(\sqrt{5}, \frac{11\pi}{3}\right)$



16. $(-\pi, -\pi)$, $(-\pi, \pi)$
$(\pi, -2\pi)$, $(\pi, 2\pi)$



17. $\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$
18. $\left(1, \sqrt{3}\right)$
19. $\left(-\frac{11\sqrt{3}}{2}, \frac{11}{2}\right)$
20. $(20, 0)$
21. $\left(0, \frac{3}{5}\right)$
22. $\left(2\sqrt{3}, -2\right)$
23. $(0, -9)$
24. $\left(-\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$
25. $\left(21\sqrt{3}, 21\right)$
26. $(117, 0)$
27. $\left(\frac{6\sqrt{5}}{5}, \frac{12\sqrt{5}}{5}\right)$
28. $\left(\sqrt{10}, 3\sqrt{10}\right)$
29. $\left(-\frac{9}{5}, -\frac{12}{5}\right)$
30. $(3, -4)$
31. $\left(-\frac{4\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right)$
32. $\left(\frac{\sqrt{26}}{52}, -\frac{5\sqrt{26}}{52}\right)$
33. $\left(\frac{4}{5}, \frac{3}{5}\right)$
34. $\left(-\frac{2}{9}, -\frac{4\sqrt{2}}{9}\right)$
35. $\left(\frac{\pi}{\sqrt{1+\pi^2}}, \frac{\pi^2}{\sqrt{1+\pi^2}}\right)$
36. $(5, 12)$
37. $\left(5, \frac{\pi}{2}\right)$
38. $\left(2\sqrt{3}, \frac{\pi}{6}\right)$
39. $\left(7\sqrt{2}, \frac{7\pi}{4}\right)$
40. $\left(2\sqrt{3}, \frac{7\pi}{6}\right)$
41. $(3, \pi)$
42. $\left(2, \frac{3\pi}{4}\right)$
43. $\left(8, \frac{4\pi}{3}\right)$
44. $\left(\frac{1}{2}, \frac{11\pi}{6}\right)$
45. $\left(\frac{3}{5}, \frac{2\pi}{3}\right)$
46. $\left(\sqrt{10}, \frac{5\pi}{4}\right)$
47. $\left(10, \arctan\left(\frac{4}{3}\right)\right)$
48. $(5, \arctan(2))$
49. $\left(\sqrt{65}, \pi - \arctan\left(\frac{1}{8}\right)\right)$
50. $(20, \pi - \arctan(3))$
51. $\left(13, \pi + \arctan\left(\frac{12}{5}\right)\right)$
52. $\left(\frac{1}{3}, \pi + \arctan(2)\right)$
53. $\left(25, 2\pi - \arctan\left(\frac{7}{24}\right)\right)$
54. $\left(15, 2\pi - \arctan\left(\frac{3}{4}\right)\right)$
55. $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{3}\right)$
56. $\left(\sqrt{13}, \pi - \arctan(2)\right)$
57. $r = 6\sec(\theta)$
58. $r = -3\sec(\theta)$
59. $r = 7\csc(\theta)$
60. $\theta = 0$
61. $\theta = \frac{3\pi}{4}$
62. $\theta = \frac{\pi}{3}$
63. $\theta = \arctan(2)$
64. $r = 5$
65. $r = \sqrt{117}$
66. $r = \frac{19}{4\cos(\theta) - \sin(\theta)}$
67. $x = \frac{1}{\cos(\theta) - 3\sin(\theta)}$
68. $r = \frac{-\sec(\theta)\tan(\theta)}{3}$
69. $r = 4\csc(\theta)\cot(\theta)$
70. $r = 2\sin(\theta)$
71. $r = 4\cos(\theta)$
72. $r = \cos(\theta)$

73. $r = 7\sin(\theta)$
74. $r = -4\cos(\theta)$
75. $r = 6\sin(\theta)$
76. $r = \frac{1}{2}\sin(\theta)$
77. $x^2 + y^2 = 49$
78. $x^2 + y^2 = 9$
79. $x^2 + y^2 = 2$
80. $y = x$
81. $y = -\sqrt{3}x$
82. $y = 0$
83. $x = 0$
84. $x^2 + y^2 = 4x$ or $(x-2)^2 + y^2 = 4$
85. $5x^2 + 5y^2 = x$ or $\left(x - \frac{1}{10}\right)^2 + y^2 = \frac{1}{100}$
86. $x^2 + y^2 = 3y$ or $x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$
87. $x^2 + y^2 = -2y$ or $x^2 + (y+1)^2 = 1$
88. $x = 7$
89. $y = \frac{1}{12}$
90. $x = -2$
91. $y = -\sqrt{5}$
92. $x^2 = 2y$
93. $y^2 = -x$
94. $\left(x^2 + y^2\right)^2 = 2xy$
95. $\left(x^2 + 2x + y^2\right)^2 = x^2 + y^2$
96. $\left(x^2 + y^2 + y\right)^2 = x^2 + y^2$
97. Any point of the form $(0, \theta)$ will work, e.g. $(0, \pi), (0, -117), \left(0, \frac{23\pi}{4}\right)$ and $(0, 0)$.